Rerserving Analysis with R

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Abstract

This paper demonstrates the `reserving` package in R [5] by examples. The `reserving` package contains implementations of the results of Mack [2, 3], Quarg [4] and England and Verrall [1].

1 The `reserving`-Package

1.1 Chain Ladder

```
> library(reserving)
> data(GenIns)
> GenIns

       V1      V2      V3      V4      V5      V6      V7      V8      V9      V10
1  357848 1124788 1735330 2218270 2745596 3319994 3466336 3606286 3833515  3901463
2  352118 1236139 2170033 3353322 3799067 4120063 4647867 4914039 5339085       NA
3  290507 1292306 2218525 3235179 3985995 4132918 4628910 4909315       NA       NA
4  310608 1418858 2195047 3757447 4029929 4381982 4588268       NA       NA       NA
5  443160 1136350 2128333 2897821 3402672 3873311       NA       NA       NA       NA
6  396132 1333217 2180715 2985752 3691712       NA       NA       NA       NA       NA
7  440832 1288463 2419861 3483130       NA       NA       NA       NA       NA       NA
8  359480 1421128 2864498       NA       NA       NA       NA       NA       NA       NA
9  376686 1363294       NA       NA       NA       NA       NA       NA       NA       NA
10 344014       NA       NA       NA       NA       NA       NA       NA       NA       NA

> G <- ChainLadder(GenIns, quarterly = FALSE, usetail = FALSE,
+     YOA = c(1997:2006))
> G

Call:
ChainLadder(triangle = GenIns, usetail = FALSE, quarterly = FALSE)

Tail-Factor:
[1] 1

by YOA:
       Latest: CL-Dev.%: CL-Ultimate: CL-Reserve: Mack S.E.: S.E.Res.%:
1997 3,901,463    100.00    3,901,463           0          0        NaN
1998 5,339,085     98.26    5,433,719      94,634     75,535       79.8
1999 4,909,315     91.27    5,378,826     469,511    121,699       25.9
2000 4,588,268     86.61    5,297,906     709,638    133,549       18.8
2001 3,873,311     79.73    4,858,200     984,889    261,406       26.5
2002 3,691,712     72.23    5,111,171   1,419,459    411,010       29.0
2003 3,483,130     61.53    5,660,771   2,177,641    558,317       25.6
2004 2,864,498     42.22    6,784,799   3,920,301    875,328       22.3
2005 1,363,294     24.16    5,642,266   4,278,972    971,258       22.7
2006   344,014      6.92    4,969,825   4,625,811  1,363,155       29.5

                          Totals:
Sum of Latest:          34,358,090
Sum of CL-Ultimate:     53,038,946
Sum of CL-Reserve:      18,680,856
Total Mack S.E.:         2,447,095
Total S.E.% of Reserve:         13

> G$Ratios
```

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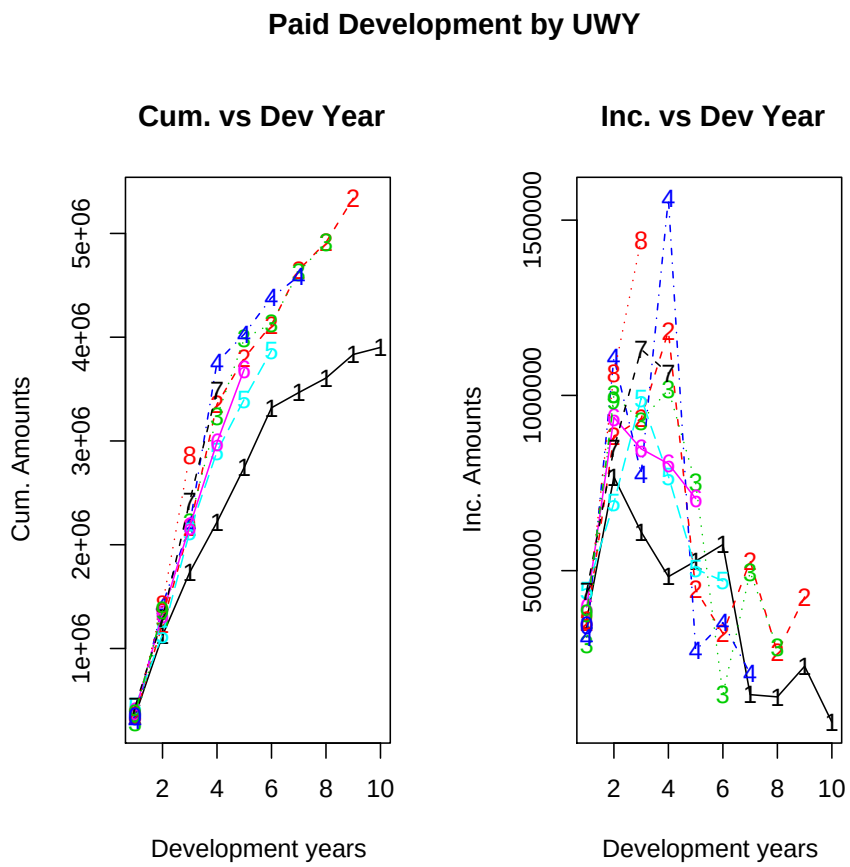
```
 [1] 3.490607 1.747333 1.457413 1.173852 1.103824 1.086269 1.053874 1.076555
 [9] 1.017725 1.000000
```

```
> G$sigma^2/1000
```

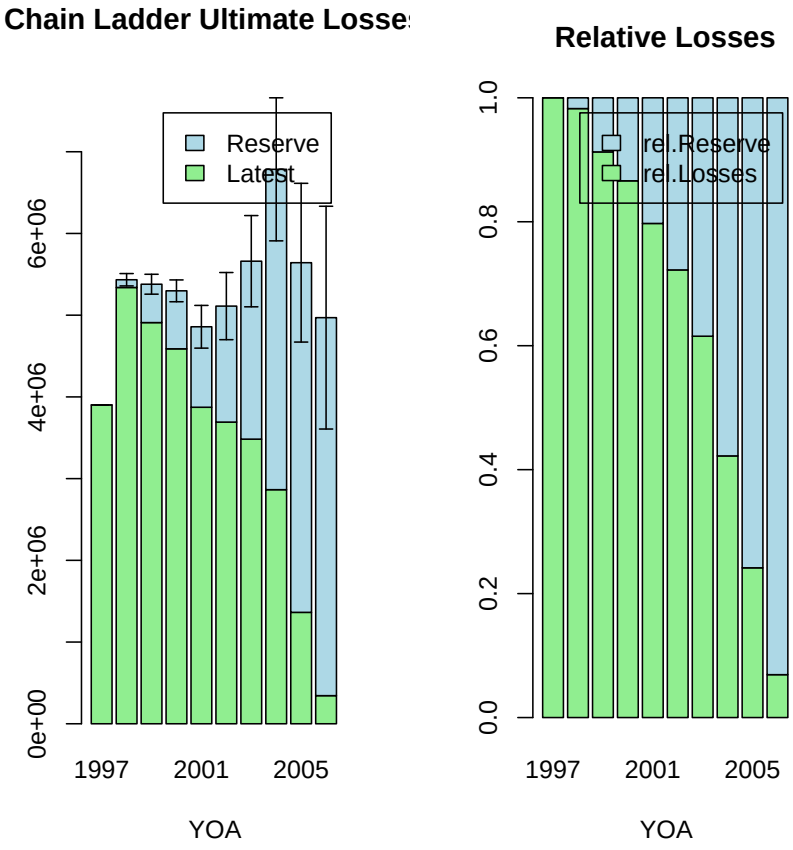
```
 [1] 160.2803275  37.7368550  41.9652130  15.1829027  13.7313239   8.1857716
 [7]   0.4466166   1.1473660   0.4466166   0.0000000
```

```
> par(mfrow = c(1, 2), oma = c(0, 0, 2, 0))
> matplot(t(GenIns), t = "o", main = "Cum. vs Dev Year", xlab = "Development years",
+     ylab = "Cum. Amounts")
> matplot(t(incTria(GenIns)), t = "o", main = "Inc. vs Dev Year",
+     xlab = "Development years", ylab = "Inc. Amounts")
> title("Paid Development by UWY", out = TRUE)
```

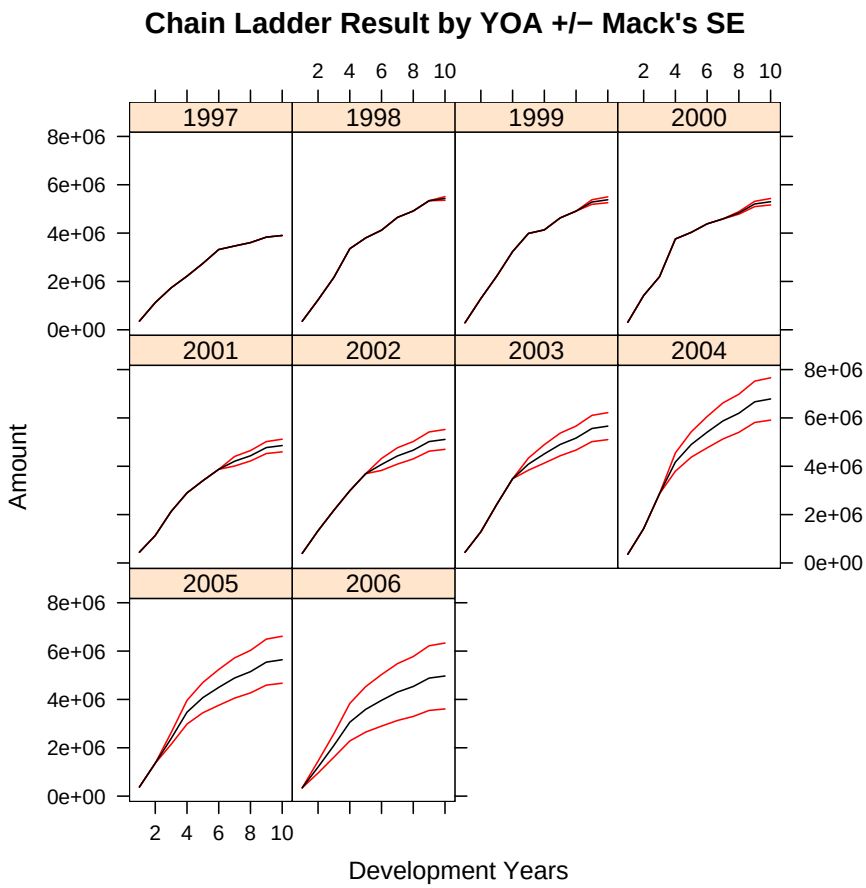



```
> par(mfrow = c(1, 1))
```

```
> plot(G)
```



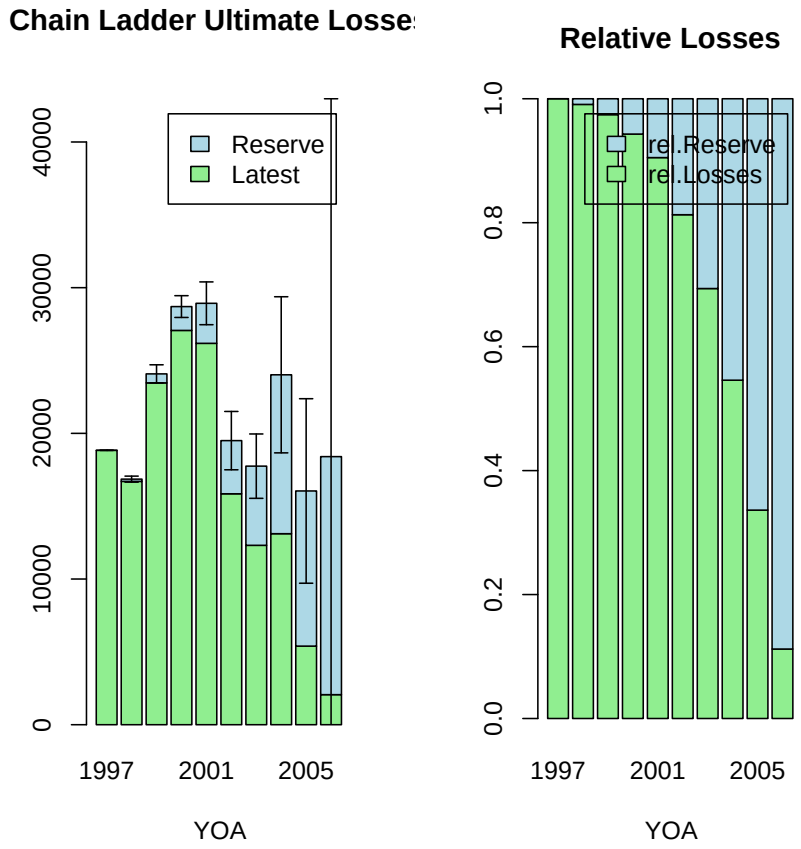

```
> plot(G, byYOA = TRUE)
```



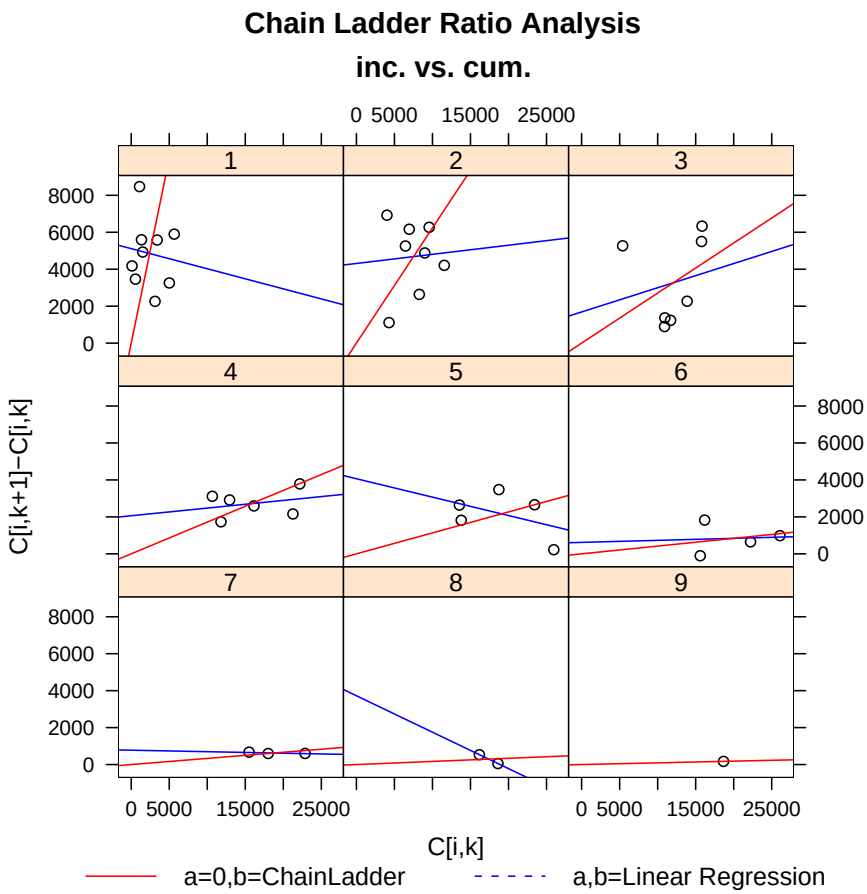

```
> data(Mortgage)
> Mortgage

     V1     V2      V3      V4      V5      V6      V7      V8      V9
1 58046 127970  476599 1027692 1360489 1647310 1819179 1906852 1950105
2 24492 141767  984288 2142656 2961978 3683940 4048898 4115760      NA
3 32848 274682 1522637 3203427 4445927 5158781 5342585      NA      NA
4 21439 529828 2900301 4999019 6460112 6853904      NA      NA      NA
5 40397 763394 2920745 4989572 5648563      NA      NA      NA      NA
6 90748 951994 4210640 5866482      NA      NA      NA      NA      NA
7 62096 868480 1954797      NA      NA      NA      NA      NA      NA
8 24983 284441      NA      NA      NA      NA      NA      NA      NA
9 13121     NA      NA      NA      NA      NA      NA      NA      NA

> M <- ChainLadder(Mortgage, quarterly = FALSE, usetail = FALSE)
> M

Call:
ChainLadder(triangle = Mortgage, usetail = FALSE, quarterly = FALSE)

Tail-Factor:
[1] 1

by YOA:
    Latest: CL-Dev.%: CL-Ultimate: CL-Reserve: Mack S.E.: S.E.Res.%:
1 1,950,105    100.00    1,950,105           0          0        NaN
2 4,115,760     97.78    4,209,118      93,358     60,883       65.2
3 5,342,585     95.27    5,607,658     265,073    139,670       52.7
4 6,853,904     89.15    7,688,163     834,259    319,020       38.2
5 5,648,563     78.28    7,216,272   1,567,709    596,210       38.0
6 5,866,482     61.35    9,562,602   3,696,120  1,037,862       28.1
7 1,954,797     35.92    5,442,091   3,487,294  1,298,251       37.2
8   284,441      8.78    3,240,567   2,956,126  1,806,032       61.1
9    13,121      0.79    1,659,913   1,646,792  2,182,258      132.5

                           Totals:
Sum of Latest:          32,029,758
```

```
Sum of CL-Ultimate:     46,576,488
Sum of CL-Reserve:      14,546,730
Total Mack S.E.:         3,728,870
Total S.E.% of Reserve:         26

> M$Ratios

[1] 11.104259  4.092273  1.707913  1.275920  1.138912  1.068697  1.026335
[8]  1.022683  1.000000

> M$sigma^2/1000

[1] 1787.4846822  977.0856457  193.7229652   42.8428360   26.9615689
[6]    5.5654230    1.2597642    0.2851546    0.0000000

> MT <- ChainLadder(Mortgage, quarterly = FALSE, tail = 1.05)
> MT

Call:
ChainLadder(triangle = Mortgage, tail = 1.05, quarterly = FALSE)

Tail-Factor:
[1] 1.05

by YOA:
    Latest: CL-Dev.%: CL-Ultimate: CL-Reserve: Mack S.E.: S.E.Res.%:
1 1,950,105    95.238    2,047,610      97,505     53,141       54.5
2 4,115,760    93.126    4,419,573     303,813    131,311       43.2
3 5,342,585    90.736    5,888,041     545,456    211,798       38.8
4 6,853,904    84.904    8,072,571   1,218,667    395,092       32.4
5 5,648,563    74.548    7,577,086   1,928,523    656,179       34.0
6 5,866,482    58.427   10,040,732   4,174,250  1,120,477       26.8
7 1,954,797    34.209    5,714,195   3,759,398  1,371,207       36.5
8   284,441     8.360    3,402,595   3,118,154  1,898,388       60.9
9    13,121     0.753    1,742,908   1,729,787  2,291,818      132.5

                          Totals:
Sum of Latest:          32,029,758
Sum of CL-Ultimate:     48,905,313
Sum of CL-Reserve:      16,875,555
Total Mack S.E.:         4,030,627
Total S.E.% of Reserve:         24

> MT$Ratios

[1] 11.104259  4.092273  1.707913  1.275920  1.138912  1.068697  1.026335
[8]  1.022683  1.050000

> MT$sigma

[1] 1336.96847  988.47643  440.13971  206.98511  164.19978   74.60176   35.49316
[8]   16.88652    0.00000

> data(RAA)
> R <- ChainLadder(RAA, quarterly = FALSE, YOA = 1997:2006)
> R

Call:
ChainLadder(triangle = RAA, quarterly = FALSE)

Tail-Factor:
[1] 1

by YOA:
     Latest: CL-Dev.%: CL-Ultimate: CL-Reserve: Mack S.E.: S.E.Res.%:
1997  18,834     100.0       18,834           0          0        NaN
1998  16,704      99.1       16,858         154        206      133.9
1999  23,466      97.4       24,083         617        623      101.0
2000  27,067      94.3       28,703       1,636        747       45.7
2001  26,180      90.5       28,927       2,747      1,469       53.5
2002  15,852      81.3       19,501       3,649      2,002       54.9
2003  12,314      69.4       17,749       5,435      2,209       40.6
2004  13,112      54.6       24,019      10,907      5,358       49.1
2005   5,395      33.6       16,045      10,650      6,333       59.5
2006   2,063      11.2       18,402      16,339     24,566      150.3

                        Totals:
Sum of Latest:          160,987
Sum of CL-Ultimate:     213,122
Sum of CL-Reserve:       52,135
Total Mack S.E.:         26,909
Total S.E.% of Reserve:      52
```

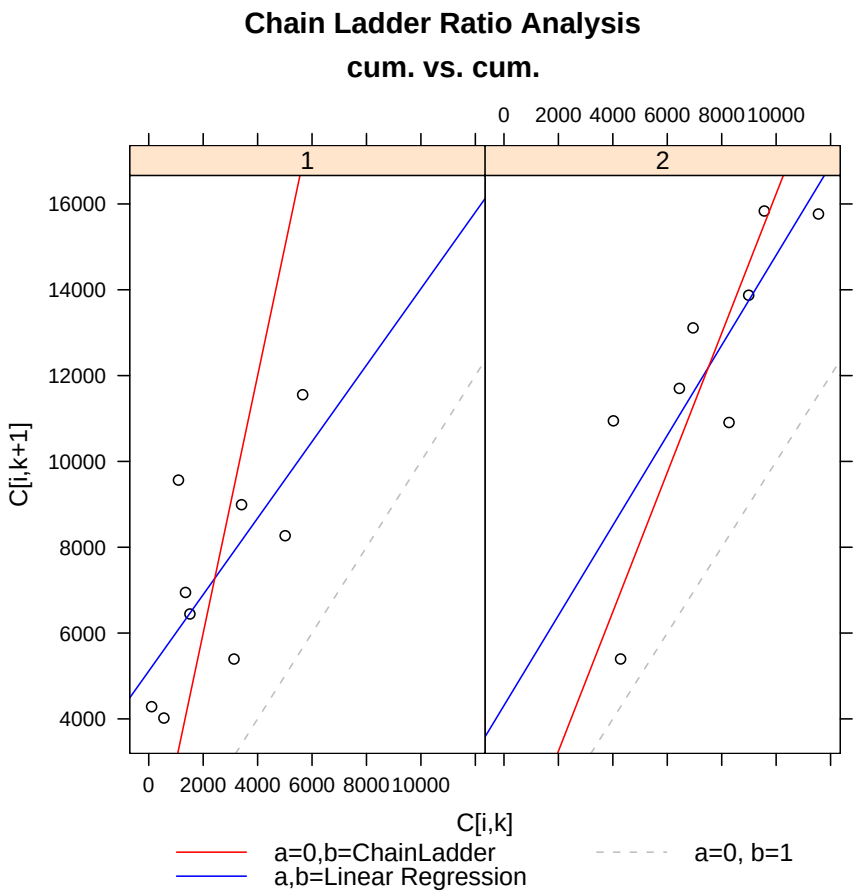
```
> plot(R)
```



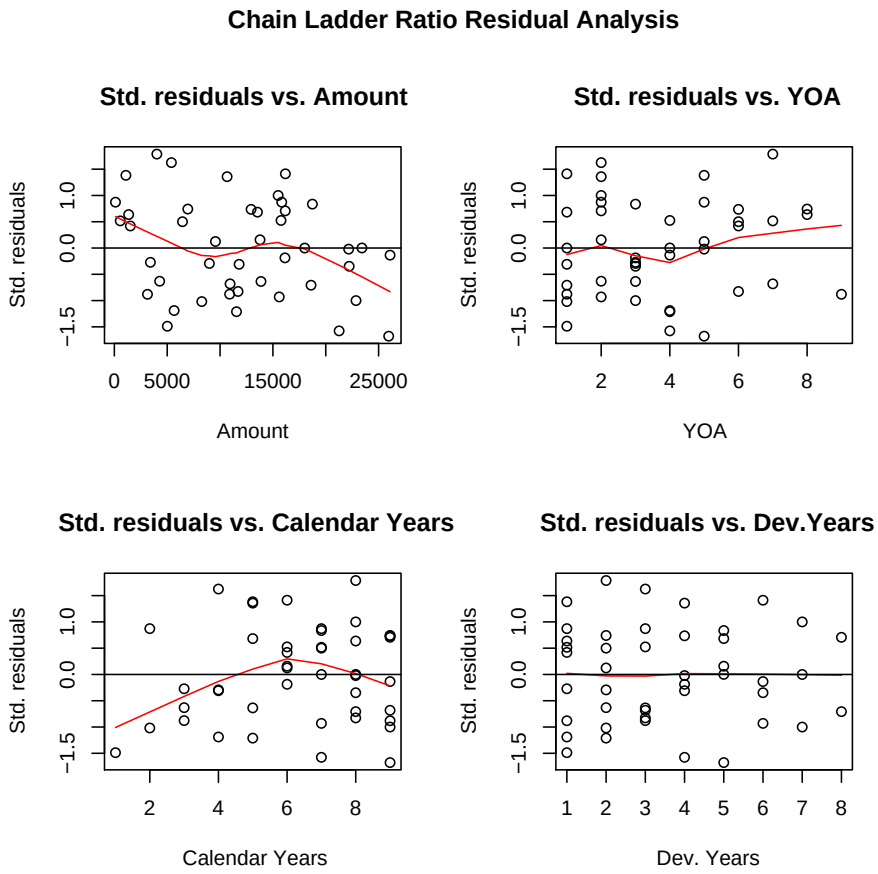

```
> plotCLRatios(R, inc.vs.cum = TRUE)
```



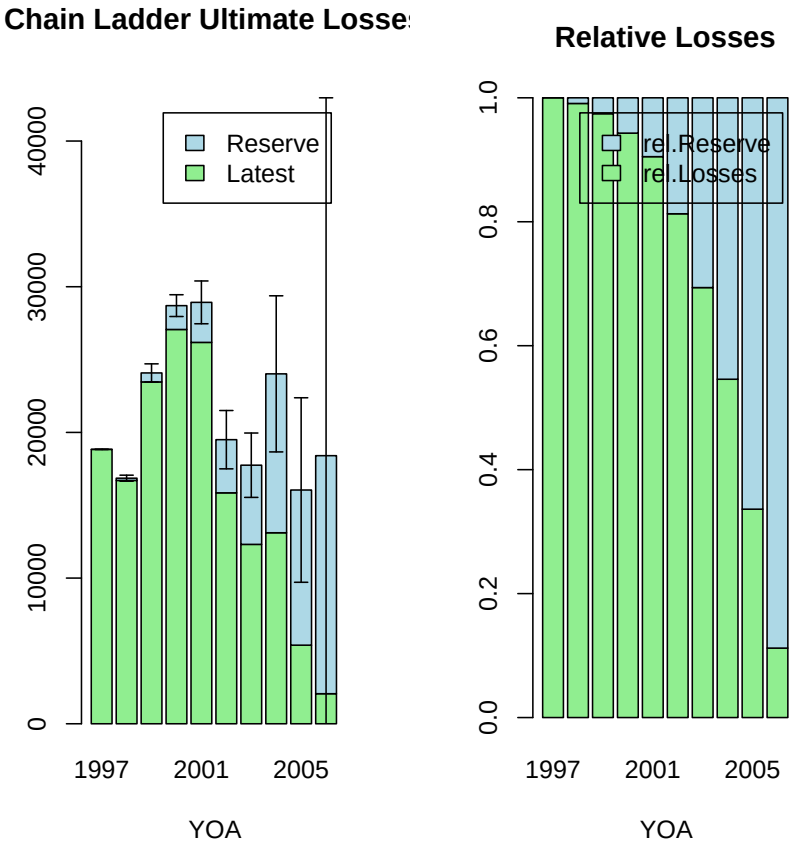

```
> plotCLRatios(R, inc.vs.cum = FALSE, dev = 1:2)
```



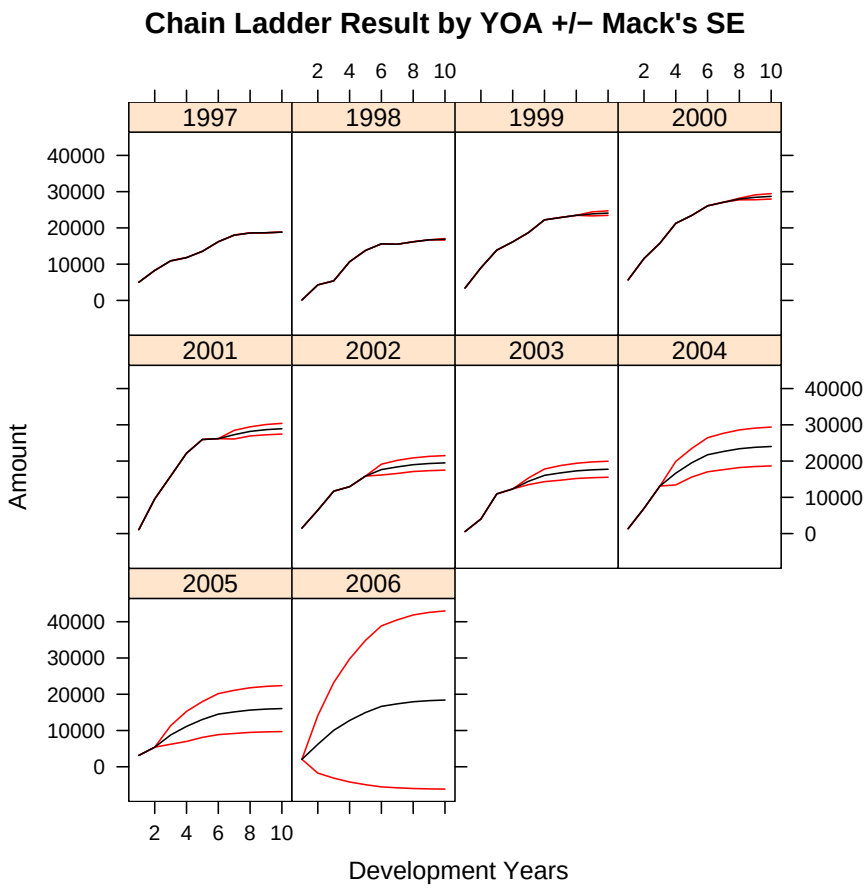

```
> plotCLResiduals(R)

        The following object(s) are masked from package:stats :

         residuals
```


Chain Ladder Ratio Residual Analysis
Std. residuals vs. Amount
Std. residuals
Amount
0
5000
15000
25000
Std. residuals vs. YOA
YOA
2
4
6
8
Std. residuals vs. Calendar Years
Calendar Years
Std. residuals vs. Dev.Years
Dev. Years
1
2
3
4
5
6
7
8
−1.5
0.0
1.0

```
> plot(R)
```



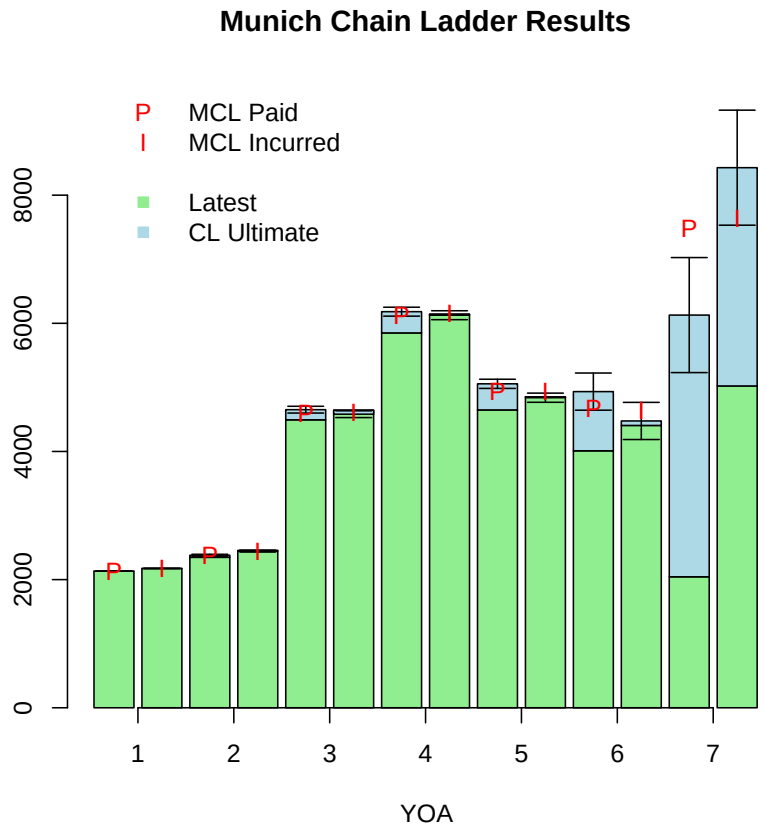

```
> plot(R, byYOA = TRUE)
```



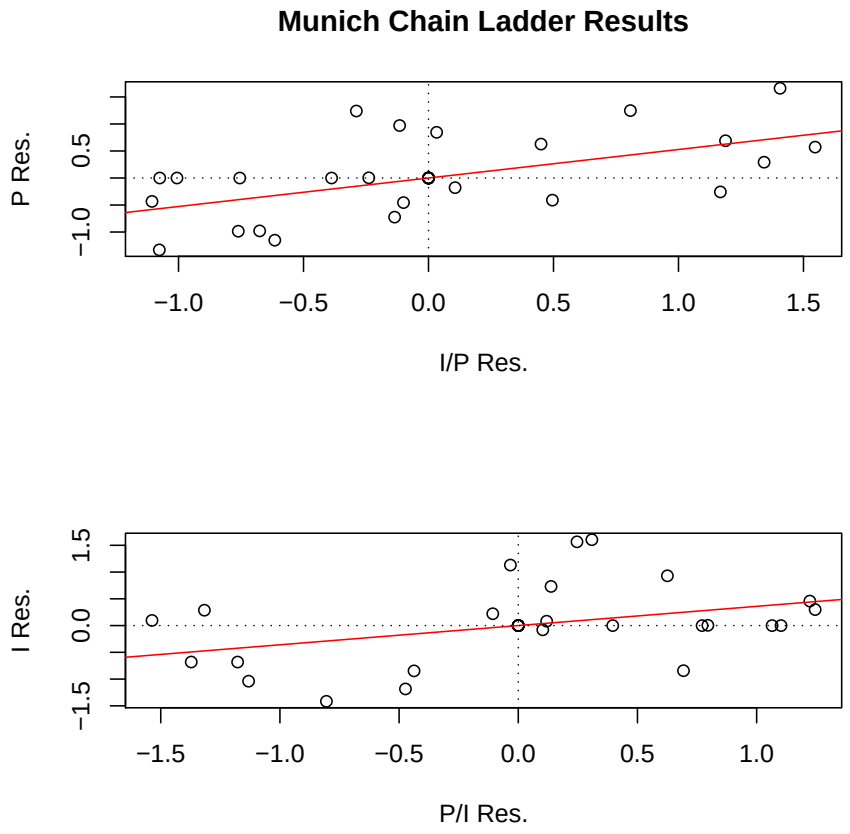
1.2 Munich Chain Ladder

```
> data(MCLpaid)
> MCLpaid

    V1   V2   V3   V4   V5   V6   V7
1  576 1804 1970 2024 2074 2102 2131
2  866 1948 2162 2232 2284 2348   NA
3 1412 3758 4252 4416 4494   NA   NA
4 2286 5292 5724 5850   NA   NA   NA
5 1868 3778 4648   NA   NA   NA   NA
6 1442 4010   NA   NA   NA   NA   NA
7 2044   NA   NA   NA   NA   NA   NA

> data(MCLincurred)
> MCLincurred

    V1   V2   V3   V4   V5   V6   V7
1  978 2104 2134 2144 2174 2182 2174
2 1844 2552 2466 2480 2508 2454   NA
3 2904 4354 4698 4600 4644   NA   NA
4 3502 5958 6070 6142   NA   NA   NA
5 2812 4882 4852   NA   NA   NA   NA
6 2642 4406   NA   NA   NA   NA   NA
7 5022   NA   NA   NA   NA   NA   NA

> M = MunichChainLadder(MCLpaid, MCLincurred, quarterly = FALSE)
> M

Call:
MunichChainLadder(paid = MCLpaid, incurred = MCLincurred, quarterly = FALSE)

Tail-Factor: Paid, Incurred
[1] 1 1
Correlation: lambdaP, lambdaI
```

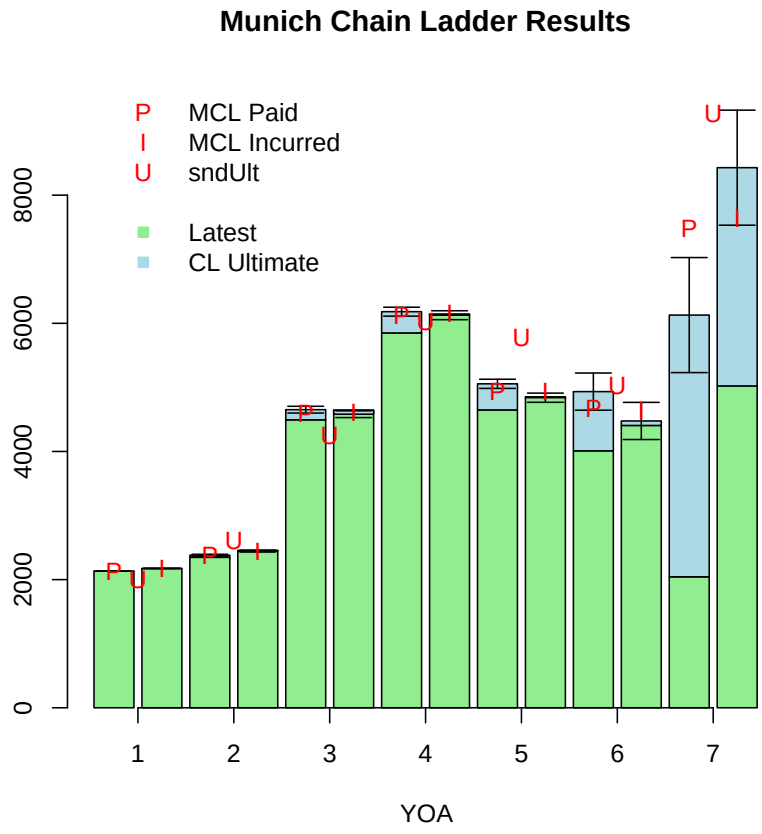
```
[1] 0.3885642 0.5158410
by YOA:
  Latest Paid: Latest Incurred: MCL-Paid: MCL-Incurred: MCL(P/I)%:
1        2,131            2,174     2,131         2,174       98.0
2        2,348            2,454     2,383         2,443       97.6
3        4,494            4,644     4,585         4,633       99.0
4        5,850            6,142     6,107         6,187       98.7
5        4,648            4,852     4,918         4,963       99.1
6        4,010            4,406     4,676         4,713       99.2
7        2,044            5,022     7,191         7,331       98.1

                              Totals:
Sum of Latest Paid:            25,525
Sum of Latest Incurred:        25,525
Sum of MCL-Ultimate Paid:      31,991
Sum of MCL-Ultimate Incurred:  32,444
Total MCL(P/I)%:                   99

> MCL = MunichChainLadder(MCLpaid, MCLincurred, quarterly = FALSE,
+     sigmaP = 0.1, sigmaI = 0.1)
> MCL

Call:
MunichChainLadder(paid = MCLpaid, incurred = MCLincurred, quarterly = FALSE,
    sigmaI = 0.1)

Tail-Factor: Paid, Incurred
[1] 1 1
Correlation: lambdaP, lambdaI
[1] 0.5274885 0.3592252
by YOA:
  Latest Paid: Latest Incurred: MCL-Paid: MCL-Incurred: MCL(P/I)%:
1        2,131            2,174     2,131         2,174       98.0
2        2,348            2,454     2,382         2,444       97.5
3        4,494            4,644     4,605         4,621       99.6
4        5,850            6,142     6,128         6,168       99.4
5        4,648            4,852     4,951         4,936      100.3
6        4,010            4,406     4,681         4,653      100.6
7        2,044            5,022     7,491         7,648       97.9

                              Totals:
Sum of Latest Paid:            25,525
Sum of Latest Incurred:        25,525
Sum of MCL-Ultimate Paid:      32,369
Sum of MCL-Ultimate Incurred:  32,645
Total MCL(P/I)%:                   99

> plot(MCL)
```




```
> plot(MCL, plotResiduals = TRUE)
```



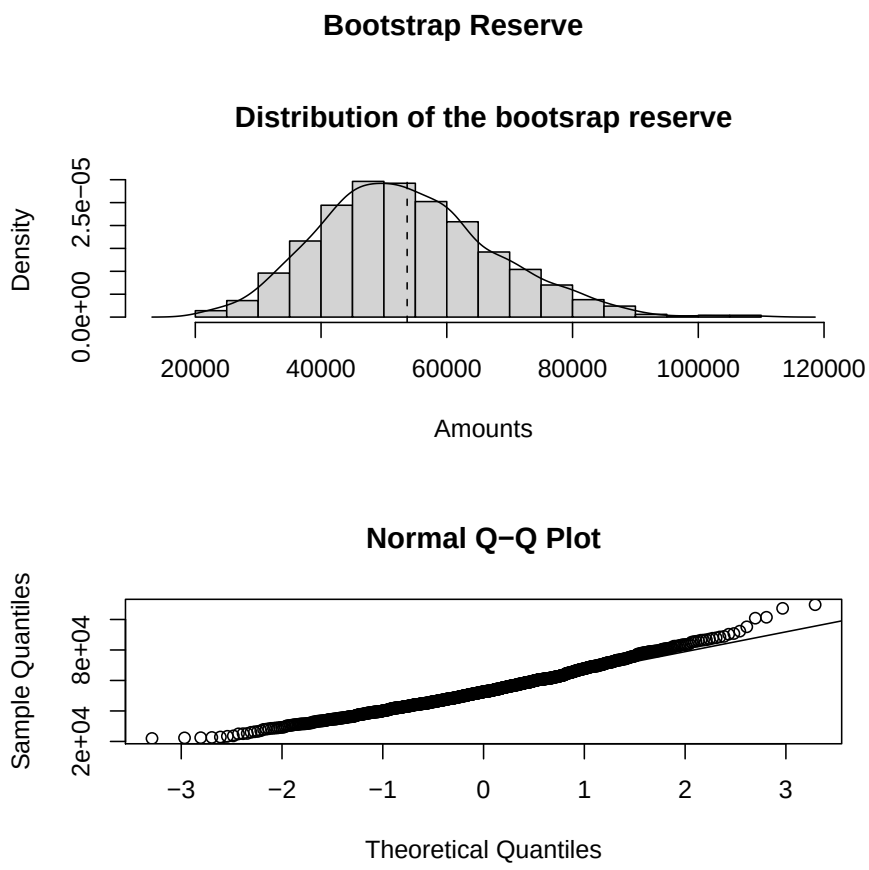

```
> U <- MCL$UltimateI * (1 + rnorm(7)/10)

> plot(MCL, sndUlt = U)
```



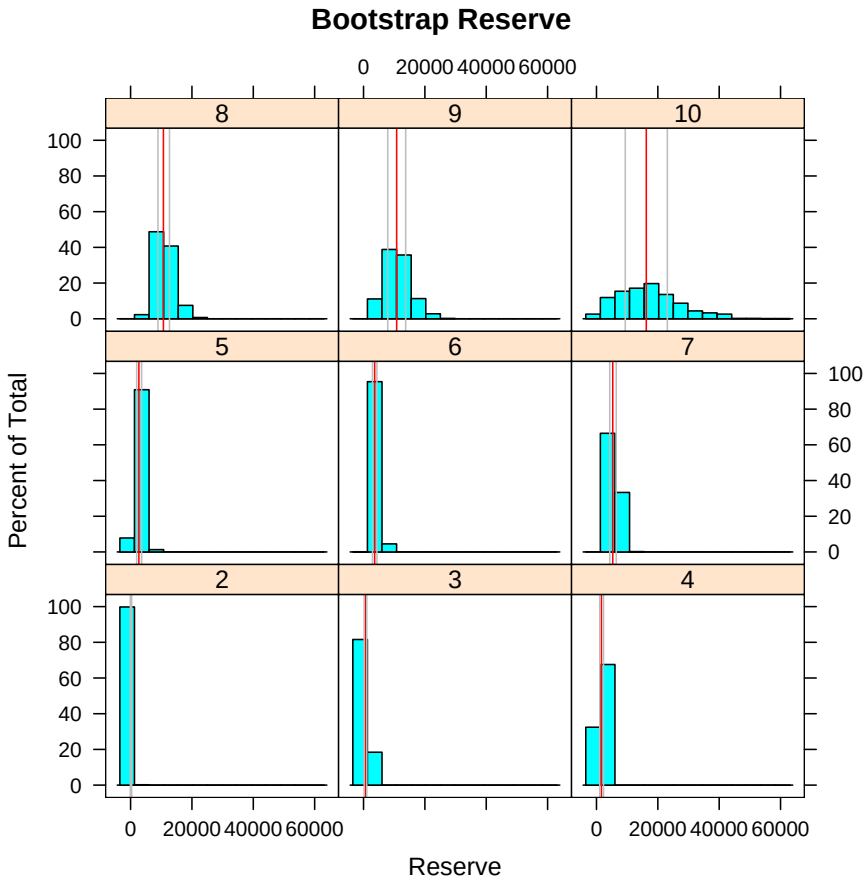
1.3 The Bootstarp

```
> data(GenIns)
> GenIns

       V1      V2      V3      V4      V5      V6      V7      V8      V9
1  357848 1124788 1735330 2218270 2745596 3319994 3466336 3606286 3833515
2  352118 1236139 2170033 3353322 3799067 4120063 4647867 4914039 5339085
3  290507 1292306 2218525 3235179 3985995 4132918 4628910 4909315      NA
4  310608 1418858 2195047 3757447 4029929 4381982 4588268      NA      NA
5  443160 1136350 2128333 2897821 3402672 3873311      NA      NA      NA
6  396132 1333217 2180715 2985752 3691712      NA      NA      NA      NA
7  440832 1288463 2419861 3483130      NA      NA      NA      NA      NA
8  359480 1421128 2864498      NA      NA      NA      NA      NA      NA
9  376686 1363294      NA      NA      NA      NA      NA      NA      NA
10 344014      NA      NA      NA      NA      NA      NA      NA      NA
       V10
1  3901463
2       NA
3       NA
4       NA
5       NA
6       NA
7       NA
8       NA
9       NA
10      NA

> bootG <- BootReserve(GenIns, quarterly = FALSE)
> bootG

Call:
BootReserve(quarterly = FALSE)

Tail-Factor:
```

```
[1] 1

by YOA:
     Latest: Boot-Ultimate: Boot-Reserve: Boot-S.E.: S.E.Res.: S.E.Res.%:
1  3,901,463      3,901,463             0          0         0        NaN
2  5,339,085      5,433,272        94,187     65,443   107,232      113.9
3  4,909,315      5,379,127       469,812    118,238   214,657       45.7
4  4,588,268      5,298,319       710,051    141,598   260,751       36.7
5  3,873,311      4,858,629       985,318    168,481   308,560       31.3
6  3,691,712      5,118,722     1,427,010    204,240   372,574       26.1
7  3,483,130      5,673,249     2,190,119    288,425   492,267       22.5
8  2,864,498      6,795,942     3,931,444    520,242   787,621       20.0
9  1,363,294      5,647,259     4,283,965    751,513 1,043,190       24.4
10   344,014      5,049,889     4,705,875  1,602,953 2,042,831       43.4

                           Totals:
Sum of Latest:          34,358,090
Sum of Boot-Ultimate:   53,155,872
Sum of Boot-Reserve:    18,797,782
Boot total S.E.:         2,359,461
Reserve total S.E.:      3,081,277
Total S.E.% of Reserve:         16
Quantiles total reserve:
      5%      25%      50%      75%      95%    99.5%
15210362 17108322 18732878 20265030 22856147 25714931

> data(RAA)
> RAA

     V1    V2    V3    V4    V5    V6    V7    V8    V9   V10
1  5012  8269 10907 11805 13539 16181 18009 18608 18662 18834
2   106  4285  5396 10666 13782 15599 15496 16169 16704    NA
3  3410  8992 13873 16141 18735 22214 22863 23466    NA    NA
4  5655 11555 15766 21266 23425 26083 27067    NA    NA    NA
5  1092  9565 15836 22169 25955 26180    NA    NA    NA    NA
6  1513  6445 11702 12935 15852    NA    NA    NA    NA    NA
7   557  4020 10946 12314    NA    NA    NA    NA    NA    NA
8  1351  6947 13112    NA    NA    NA    NA    NA    NA    NA
9  3133  5395    NA    NA    NA    NA    NA    NA    NA    NA
10 2063    NA    NA    NA    NA    NA    NA    NA    NA    NA

> bootR <- BootReserve(RAA, quarterly = FALSE)
> bootR

Call:
BootReserve(quarterly = FALSE)

Tail-Factor:
[1] 1

by YOA:
   Latest: Boot-Ultimate: Boot-Reserve: Boot-S.E.: S.E.Res.: S.E.Res.%:
1   18,834         18,834             0          0         0        NaN
2   16,704         16,867           163        326       568      349.3
3   23,466         24,126           660        667     1,152      174.6
4   27,067         28,774         1,707        999     1,790      104.9
5   26,180         29,021         2,841      1,209     2,242       78.9
6   15,852         19,595         3,743      1,178     2,409       64.3
7   12,314         17,799         5,485      1,598     3,049       55.6
8   13,112         24,147        11,035      2,946     4,911       44.5
9    5,395         16,427        11,032      4,279     6,231       56.5
10   2,063         19,092        17,029     10,156    13,203       77.5

                        Totals:
Sum of Latest:          160,987
Sum of Boot-Ultimate:   214,680
Sum of Boot-Reserve:     53,693
Boot total S.E.:         13,786
Reserve total S.E.:      18,525
Total S.E.% of Reserve:      35
Quantiles total reserve:
      5%      25%      50%      75%      95%    99.5%
33265.11 44131.46 52602.82 61656.82 78829.49 92370.36

> plot(bootR)
```




```
> plot(bootR, byYOA = TRUE)
```


Bootstrap Reserve
8
9
10
5
6
7
2
3
4
Percent of Total
Reserve
0
20000
40000
60000
0
20
40
60
80
100

References

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